FINANCIAL MATHEMATICS

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Ex 7.1 Pg 7.6

Q1 Find the present value of a sequence of payments of ₹ 80 made at the end of each 6 months and continuing forever, if money is worth 4% compounded semi annually

Solution

$$i = \frac{4}{2 \times 100} = \frac{2}{100} = 0.02$$

$$P = \frac{R}{i}$$

$$P = \frac{80}{0.02} = 4000$$

Q2 Find the present value of an annuity of ₹ 1800 made at the end of each quarter and continuing forever, if money is worth 5% compounded quarterly.

Solution

$$i = \frac{5\%}{4} = \frac{5}{400}, \quad R = 1800$$

$$P = \frac{R}{i} = \frac{1800}{5/400}$$

$$= 1800 \times \frac{400}{5} = 144000$$

Q3 If the cash equivalent of a perpetuity of ₹ 300 payable at the end of each quarter is ₹ 24000, find the rate of interest compounded quarterly?

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Solution $P = 24000$, $R = 300$

$$P = \frac{R}{i}$$

$$24000 = \frac{300 \times 100 \times 4}{i}$$

$$i = \cancel{80}\ \frac{300 \times 100}{24000} = \frac{5}{4} \times 4$$

$$i = 5\%$$

Q4 Find the present value of a perpetuity of £ 780 payable at the beginning of each year, if money is worth 6% effective.

Solution $i = 6\% = \frac{6}{100}$

$$R = 780$$

$$P = R + \frac{R}{i}$$

$$= R\left(1 + \frac{1}{i}\right)$$

$$= 780\left(1 + \frac{1}{6/100}\right)$$

$$= 780\left(1 + \frac{100}{6}\right)$$

$$= 780 \times \frac{106}{6}$$

$$= 13780$$

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Q5 The present value of a perpetual income of ₹ x at the end of each 6 months is ₹ 36000. Find the value of x if money is worth 6% compounded semi-annually.

Solution $R = x \quad i = \frac{6}{2\times100} = \frac{3}{100}$

$$P = \frac{R}{i}$$

$$36000 = \frac{x}{3/100}$$

$$36000 = \frac{100x}{3}$$

$$360\times3 = x$$

$$1080 = x$$

Q6 If you need ₹ 20,000 for your daughters education how much you set aside each quarter for 10 years to accumulate this amount at the rate of 6% compounded quarterly?

Solution $A = 20{,}000, \quad r = \frac{6}{4\times100} = 0.015$

$$n = 10\times4 = 40$$

$$A = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$20000 = R\left[\frac{(1.015)^{40} - 1}{0.015}\right]$$

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$$20000 \times 0.015 = R[(1.015)^{40} - 1]$$

$$300.000 = R[1.81401 - 1]$$

$$\frac{300}{0.81401} = R$$

$$368.54 \quad R$$

Q7 To save for childs education, a sinking fund is created to have 100,000₹ at the end of 25 years. How much money should be retained out of the profit each year for the sinking fund, if the investment can earn interest at the rate 4% per annum.

Solution $\quad i = 4\% = \frac{4}{100}, \quad n = 25$

$$A = 100,000$$

$$A = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$100,000 = R\left[\frac{(1+0.04)^{25} - 1}{0.04}\right]$$

$$100,000 \times \frac{4}{100} = R[(1.04)^{25} - 1]$$

$$4000 = R[2.66583 - 1]$$

$$4000 = R[1.66583]$$

$$\frac{4000}{1.66583} = R$$

$$2401.205 = R$$

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Q8 A machine costs ₹ 100,000 and its effective life is estimated to be 12 years. A sinking fund is created for replacing the machine by a new model at the end of its lifetime when its scrap realises a sum of ₹ 5000 only. Find what amount should be set aside at the end of each year, out of the profits, for the sinking fund if it accumulates at 5% effective.

Solution

$$A = 100000 - 5000$$

$$A = 95000$$

$$i = \frac{5}{100} = 0.05, \quad n = 12$$

$$A = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$95000 = R\left[\frac{(1.05)^{12} - 1}{0.05}\right]$$

$$95000 \times 0.05 = R\left[1.795863 - 1\right]$$

$$\frac{4750}{0.795863} = R$$

$$5968.41 = R$$

Q9 Suppose a machine costing ₹ 50,000 is to be replaced at the end of 10 years, at that time it will have a salvage value of ₹ 5000. In order to provide money at that time for a machine costing the same amount, a sinking

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fund is set up. The amount in the fund at that time is to be the difference between the replacement cost and salvage value. If equal payments are placed in the fund at the end of each quarter and the fund earns 8% compounded quarterly. What should each payment be?

Solution Amount needed after 10 years

$= \text{Replacement cost} - \text{Salvage cost}$

$= 50,000 - 5000$

$= 45000$

$i = \frac{8}{400} = 0.02, \quad n = 10 \times 4 = 40$

$$A = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$45000 = R\left[\frac{(1.02)^{40} - 1}{0.02}\right]$$

$$900 = R[2.2080 - 1]$$

$$\frac{900}{1.2080} = R$$

$$745.03 = R$$

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Ex 7.3

CACULATION OF EMI

Q1 Mohan takes a loan of ₹500,000 with 8% annual interest rate for 6 years. Calculate EMI under flat rate system

Solution

$P = 500,000$

$R = 8\% \qquad T = n = 6 \times 12 = 72$

$$I = \frac{8}{100} \times 500,000 \times 6$$

$$= 240000$$

$$EMI = \left(\frac{P+I}{n}\right)$$

$$= \frac{500,000 + 240000}{72}$$

$$= 10277.7$$

Q2 XYZ company borrows ₹300,000 with 7% annual interest rate for 4 years. Calculate EMI under Reducing Balance method.

Solution

$P = 300,000 \qquad t = 12$

$$i = \frac{7}{100 \times 12} = 0.0058$$

$$n = 12 \times 4 = 48$$

$$R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$= 300000\left[\frac{0.0058(1+0.0058)^{48}}{(1+0.0058)^{48} - 1}\right]$$

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$$= 300000 \times \left[\frac{0.0058 \times (1.0058)^{48}}{(1.0058)^{48} - 1}\right]$$

$$= \frac{3,00,000 \times 0.0058 \times 1.3199}{1.3199 - 1}$$

$$= \frac{3,00,000 \times 0.0058 \times 1.3199}{0.3199}$$

$$= 7179.19$$

Q3 Rajesh borrows ₹ 6,00,000 with 9% annual interest rate for 5 years. Calculate EMI under Reducing Balance method.

Solution

$$P = 600,000, \quad T = 5 \text{ years}$$

$$i = \frac{9}{100 \times 12} = \frac{3}{400} = 0.0075$$

$$n = 5 \times 12 = 60$$

$$R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$= 6,00,000\left[\frac{0.0075(1+0.0075)^{60}}{(1.0075)^{60} - 1}\right]$$

$$= 6,00,000\left[\frac{0.0075 \times (1.0075)^{60}}{(1.0075)^{60} - 1}\right]$$

$$= 6,00,000\left[\frac{0.0075 \times 1.5656}{1.5656 - 1}\right]$$

$$= \frac{6,00,000 \times 0.0075 \times 1.5656}{0.5656}$$

$$= \frac{7045.2}{0.5656} = 12456.15$$

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Q4 A person amortizes a loan of ₹ 1,50,000 for a new home by obtaining a 10 year mortage at the rate of 12% compounded monthly. find
(i) The monthly payments (ii) Total interest paid
[Given $a_{\overline{120}|0.01} = 69.6891$]

Solution (i) $P = 150,000$ time = 10 years

$i = 12\% = \frac{12}{12 \times 100} = 0.01$

$n = 12 \times 10 = 120$

$R = \frac{P}{a_{\overline{n}|i}} = \frac{150000}{a_{\overline{120}|0.01}}$

$= \frac{1,50,000}{69.6891}$

$R = 2152.42$

(ii) Interest Paid $= nR - P$
$= 2152.41 \times 120 - 150,000$
$= 258289.2 - 150,000$
$= 108290.4$

Q5 A couple wishes to purchase a house for ₹ 1,200,000 with a down payment of ₹ 2,50,000. If they can amortize the balance at 9% per annum compounded monthly for 20 years
(i) What is their monthly payment
(ii) What is the total interest paid

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$$\text{Given } a_{\overline{240}|0.0075} = 111.1449$$

Solution (i) $P = 1200000$

Down Payment = 2,50,000

Remaining Payment = 1200000 − 250000

= 950,000

Time = 20 years

$i = \frac{9}{100 \times 12} = 0.0075$

$n = 20 \times 12 = 246$

$R = \frac{P}{a_{\overline{n}|i}}$

$= \frac{950000}{a_{\overline{240}|0.0075}} = \frac{950000}{111.1449}$

$R = 8547.40$

ii) Total Interest paid $= nR - P$

$= 240 \times 8547.40 - 950{,}000$

$= 1101376.175$

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7.4 NOMINAL AND EFFECTIVE RATE OF INTEREST

Pg 7.17

Ex 7.4

Q.1 What is the effective annual rate of interest compounding equivalent to a nominal rate of interest 5% per annum compounded quarterly?

Solution

$$r = 5\% = \frac{5}{100} = 0.05$$

$$m = 4$$

$$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.05}{4}\right)^4 - 1$$

$$= (1.0125)^4 - 1$$

$$= 1.05094 - 1$$

$$r_{eff} = 0.05094$$

$$r_{eff} \text{ or } 5.09\%$$

Q.2 Which is the better investment 3% per year compounded monthly or 3.1% per year compounded quarterly?

Solution

$$r = 3\% = 0.03$$

$$m = 12$$

$$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.03}{12}\right)^{12} - 1$$

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$$= (1 + 0.0025)^{12} - 1$$
$$= (1.0025)^{12} - 1$$
$$= 1.03041 - 1$$
$$r_{eff} = 0.03041 \text{ or } 3.04\%$$

(ii) $r = 3.1\% = 0.031$, $m = 4$

$$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$
$$= \left(1 + \frac{0.031}{4}\right)^4 - 1$$
$$= (1 + 0.00775)^4 - 1$$
$$= 1.03136 - 1$$
$$= 0.03136$$
$$= 3.136\%$$

Better investment is 3.1% per year compounded quarterly.

Q3 What effective rate of interest is equivalent to nominal rate of 8% converted quarterly.

Solution $r = 8\% = \frac{8}{100} = 0.08$

$m = 4$

$$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$
$$= \left(1 + \frac{0.08}{4}\right)^4 - 1$$
$$= (1 + 0.02)^4 - 1$$
$$= (1.02)^4 - 1$$

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$= 1.0824 - 1$

$r_{eff} = 0.0824$

$= 8.24\%$

To what amount will £ 12000 accumulate in 12 years if invested at an effective rate of 5%?

on $m = 12$, $i = \frac{5}{100} = 0.05$

$\text{Amount} = P(1+i)^m$

$= 12000(1+0.05)^{12}$

$= 12000(1.05)^{12}$

$= 12000 \times 1.7958$

$\text{Amount} = 21549.6$

Which yields more interest 8% effective or 7.8% compounded semi-annually.

on effective $= 8\%$

$r = 7.8\%$

$r_{eff} = \left(1 + \frac{7.8}{100 \times 2}\right)^2 - 1$

$= (1 + 0.039)^2 - 1$

$= 1.079221 - 1$

$= 0.079221 \times 100\%$

$r_{eff} = 7.9\%$

COMPOUND ANNUAL GROWTH RATE (7.5)

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Ex 7.5 Pg 7.19

Q1 An investment has starting value of ₹ 5000 and it grows to ₹ 25,000 in 4 years. What will be its CAGR?

Solution $SV = 5000$, $EV = 25000$

$n = 4$

$$CAGR = \left[\left(\frac{EV}{SV}\right)^{1/n} - 1\right] \times 100$$

$$= \left[\left(\frac{25000}{5000}\right)^{1/4} - 1\right] \times 100$$

$$= [1.4953 - 1] \times 100$$

$$= 0.4953 \times 100$$

$$CAGR = 49.53\%$$

Q2 An investment has a starting value of ₹ 2000 and it grows to 18000 in 3 years. What will be its CAGR?

Solution $SV = 2000$, $EV = 18000$

$T = 3 \text{ years} = n$

$$CAGR = \left[\left(\frac{EV}{SV}\right)^{1/n} - 1\right] \times 100$$

$$= \left[\left(\frac{18000}{2000}\right)^{1/3} - 1\right] \times 100$$

$$= [9^{1/3} - 1] \times 100$$

$$= [2.08006 - 1] \times 100$$

$$= 1.08006 \times 100$$

$$= 108.006\%$$

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Q.3 Calculate CAGR from the following data

Year	2015	2016	2017	2018
Revenue(₹)	3,00,000	3,50,000	4,00,000	4,50,000

Solution $SV = 3,00,000$

$EV = 4,50,000$, $n = 3$

$$CAGR = \left[\left(\frac{EV}{SV}\right)^{1/n} - 1\right] \times 100$$

$$= \left[\left(\frac{450000}{300000}\right)^{1/3} - 1\right] \times 100$$

$$= \left[(1.5)^{1/3} - 1\right] \times 100$$

$$= [1.14471 - 1] \times 100$$

$$= 0.14471 \times 100$$

$$CAGR = 14.471\ \%$$

Q4 Mr Kumar has invested ₹ 20,000 in year 2014 for 5 years. If CAGR for that investment turned out to be 11.84%, what will be the end balance

Solution $SV = 20,000$, $n = 5$

$CAGR = 11.84$

$$CAGR = \left[\left(\frac{EV}{SV}\right)^{1/n} - 1\right] \times 100$$

$$11.84 = \left[\left(\frac{EV}{20,000}\right)^{1/5} - 1\right] \times 100$$

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$$0.1184 = \left[\left(\frac{EV}{20000}\right)^{1/5} - 1\right]$$

$$0.1184 + 1 = \left(\frac{EV}{20000}\right)^{1/5}$$

$$(1.1184)^5 = \frac{EV}{20000}$$

$$(1.1184)^5 \times 20000 = EV$$

$$1.749789444 \times 20000$$

$$34995.78 = EV$$

$$\approx 35000 = EV$$

Q5 Mr Naresh has bought 200 shares of City Look Company at ₹100 each in 2015. After selling them he has received ₹30,000 which accounts for 22.47% CAGR. Calculate the number of years for which he was holding the shares.

Solution

1 share cost = 100₹

200 shares = 100×200

SV = 20,000

EV = 30,000 , n = ?

CAGR = 22.47%

$$CAGR = \left[\left(\frac{EV}{SV}\right)^{1/n} - 1\right] \times 100$$

$$22.47 = \left[\left(\frac{30,000}{20,000}\right)^{1/n} - 1\right] \times 100$$

$$\frac{22.47}{100} = \left[\left(\frac{3}{2}\right)^{1/n} - 1\right]$$

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$$0.2247 = (1.5)^{1/n} - 1$$

$$0.2247 + 1 = (1.5)^{1/n}$$

$$1.2247 = (1.5)^{1/2}.$$

$$\log 1.2247 = \frac{1}{n} \log 1.5$$

$$n = \frac{\log 1.5}{\log 1.2247}$$

$$= \frac{0.1760912}{0.0880297}$$

$$= 2.00036$$

$$\approx 2 \text{ years}$$

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Ex 7.7

DEPRECIATION

Q1 A machine costing ₹ 30000 is expected to have a useful life of 13 years and a final scrap value of ₹ 4000. Find the annual depreciation charge using the straight line method

Solution

$C = ₹\ 30000$
$S = ₹\ 4000$
$n = 13$ years

$$D = \frac{C-S}{n} = \frac{30000-4000}{13}$$
$$= \frac{26000}{13} = 2000$$

Annual Depreciation = 2000 ₹

Q2 An asset costing ₹ 15000 is expected to have a useful life of 5 years and a scrap value of ₹ 3000. Find the annual depreciation charge using the straight line method

Solution

$C = ₹\ 15000$
$S = ₹\ 3000$
$n = 5$

$$D = \frac{C-S}{n}$$
$$= \frac{15000-3000}{5}$$
$$= \frac{12000}{5} = 2400$$

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Annual Depreciation = ₹ 2400

A firm bought a machinery for ₹ 7,40,000 on 1st April 2018 and ₹ 60,000 is spent on its installation. Its useful life is estimated to be of 5 years. Its estimated reliable or scrap value at the end of the period was estimated at ₹ 40,000. Find out the amount of annual depreciation and rate of depreciation?

Solution

$$\text{Annual depreciation} = \frac{C-S}{n}$$

$$= \frac{(\text{Cost} + \text{Erection charges}) - \text{Scrap value}}{\text{Expected useful life}}$$

$$= \frac{740000 + 60,000 - 40000}{5}$$

$$= \frac{760000}{5}$$

$$= 152000$$

D = ₹ 152000 p.a

$$\text{Rate of Depreciation} = \frac{D}{C+S}$$

$$= \frac{152000}{800000}$$

$$= 0.19$$

Rate of Depreciation = 19%

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Q7 Shiv & Co. purchased a mobile phone for ₹ 21,000 on 1 April 2019. The estimated life of the mobile phone is 10 years, after which its residual value will be ₹ 1000 only. Find out the amount of annual depreciation according to linear method

Solution

$C = ₹\ 21000$

$S = ₹\ 1000$

$n = 10$ years

$$D = \frac{C-S}{n}$$

$$= \frac{21000 - 1000}{10}$$

$$D = \frac{20,000}{10} = 2000 ₹\ p.a$$

Q8 On 1st April 2015, Dreams Ltd purchased an AC for ₹ 3,00,000 and incurred ₹ 21000 towards freight ₹ 3000 towards carriage and ₹ 6000 towards installation charges. It has been estimated that the machinery will have a scrap value of ₹ 30,000 at the end of the useful life which is four years. What will be the annual depreciation and the value of machinery after four years according to linear method.

Solution-

$$D = \frac{C-S}{n}$$

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$S = 30{,}000 \quad , \quad n=4$

$$D = \frac{300000 + 21000 + 3000 + 6000 - 30{,}000}{4}$$

$$= \frac{300000}{4} = 75000$$

$$D = ₹\ 75000 \text{ p.a}$$

Depreciation schedule

Year	Depreciation	Accumulated Depreciation	Book value of the asset
0	0	0	3,30,000
1	75000	75000	255000
2	75000	150000	180000
3	75000	225000	105000
4	75000	300000	30,000

Value of the machinery after 4 years = ₹ 30,000